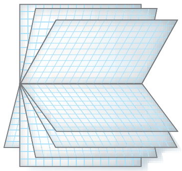
CHAPTER 7 Study Guide and Review

Download Vocabulary Review from algebra2.com

FOLDABLES™ Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Operations on Functions (Lesson 7-1)

Operation	Definition
Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
Composition	$[f \circ g](x) = f[g(x)]$

Inverse and Square Root Functions (Lesson 7-2 and 7-3)

- Reverse the coordinates of ordered pairs to find the inverse of a relation.
- Two functions are inverses if and only if both of their compositions are the identity function.

Roots of Real Numbers (Lesson 7-4)

Real *n*th roots of b, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$			
n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$\sqrt[n]{b}$ if $b = 0$
even	one positive root one negative root	no real roots	one real root, 0
odd	one positive root no negative roots	no positive roots one negative root	one real root, 0

Radicals (Lessons 7-5 through 7-7)

For any real numbers a and b and any integer $n > 1$,

- Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$.
- To solve a radical equation, isolate the radical. Then raise each side of the equation to a power equal to the index of the radical.

Key Vocabulary

composition of functions (p. 385)
conjugates (p. 411)
extraneous solution (p. 422)
identity function (p. 393)
inverse function (p. 392)
inverse relation (p. 391)
like radical expressions (p. 411)
*n*th root (p. 402)
one-to-one (p. 394)
principal root (p. 402)
radical equation (p. 422)
radical inequality (p. 424)
rationalizing the denominator (p. 409)
square root function (p. 397)
square root inequality (p. 399)

Vocabulary Check

Choose a word or term from the list above that best completes each statement or phrase.

1. A(n)_______ is an equation with radicals that have variables in the radicands.
2. A solution of a transformed equation that is not a solution of the original equation is a(n) _______.
3. _______ have the same index and the same radicand.
4. When a number has more than one real root, the _______ is the nonnegative root.
5. $f(x) = 6x - 2$ and $g(x) = \frac{x + 2}{6}$ are _______ since $[f \circ g](x) = x$ and $[g \circ f](x) = x$.
6. A(n) _______ is when a function is performed, and then a second function is performed on the result of the first function.
7. A(n) _______ function is a function whose inverse is a function.
8. The process of eliminating radicals from a denominator or fractions from a radicand is called _______.
9. Two relations are _______ if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).

Vocabulary Review at algebra2.com

Lesson-by-Lesson Review

7–1 Operations on Functions (pp. 384–390)

Find $[g \circ h](x)$ and $[h \circ g](x)$.

10. $h(x) = 2x - 1$, $g(x) = 3x + 4$

11. $h(x) = x^2 + 2$, $g(x) = x - 3$

12. $h(x) = x^2 + 1$, $g(x) = -2x + 1$

13. $h(x) = -5x$, $g(x) = 3x - 5$

14. $h(x) = x^3$, $g(x) = x - 2$

15. $h(x) = x + 4$, $g(x) = |x|$

16. **TIME** The formula $h = \frac{m}{60}$ converts minutes m to hours h, and $d = \frac{h}{24}$ converts hours to days d. Write a composition of functions that converts minutes to days.

Example 1 **If $f(x) = x^2 - 2$ and $g(x) = 8x - 1$, find $g[f(x)]$ and $f[g(x)]$.**

$g[f(x)] = 8(x^2 - 2) - 1$ Replace $f(x)$ with $x^2 - 2$.

$= 8x^2 - 16 - 1$ Multiply.

$= 8x^2 - 17$ Simplify.

$f[g(x)] = (8x - 1)^2 - 2$ Replace $g(x)$ with $8x - 1$.

$= 64x^2 - 16x + 1 - 2$ Expand.

$= 64x^2 - 16x - 1$ Simplify.

7–2 Inverse Functions and Relations (pp. 391–396)

Find the inverse of each function. Then graph the function and its inverse.

17. $f(x) = 3x - 4$

18. $f(x) = -2x - 3$

19. $g(x) = \frac{1}{3}x + 2$

20. $f(x) = \frac{-3x + 1}{2}$

21. $y = x^2$

22. $y = (2x + 3)^2$

23. **SALES** Jim earns \$10 an hour plus a 10% commission on sales. Write a function to describe Jim's income. If Jim wants to earn \$1000 in a 40-hour week, what should his sales be?

24. **BANKING** During the last month, Jonathan has made two deposits of \$45, made a deposit of double his original balance, and withdrawn \$35 five times. His balance is now \$189. Write an equation that models this problem. How much money did Jonathan have in his account at the beginning of the month?

Example 2 **Find the inverse of $f(x) = -3x + 1$.**

Rewrite $f(x)$ as $y = -3x + 1$. Then interchange the variables and solve for y.

$x = -3y + 1$ Interchange the variables.

$3y = -x + 1$ Solve for y.

$y = \frac{-x + 1}{3}$ Divide each side by 3.

$f^{-1}(x) = \frac{-x + 1}{3}$ Rewrite in function notation.

Study Guide and Review

7-3 Square Root Functions and Inequalities (pp. 397–401)

Graph each function.

25. $y = \frac{1}{3}\sqrt{x+2}$

26. $y = \sqrt{5x-3}$

27. $y = 4 + 2\sqrt{x-3}$

Graph each inequality.

28. $y \geq \sqrt{x} - 2$

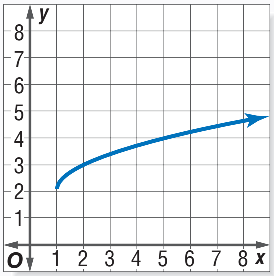
29. $y < \sqrt{4x-5}$

30. OCEAN The speed a tsunami, or tidal wave, can travel is modeled by the equation $s = 356\sqrt{d}$, where s is the speed in kilometers per hour and d is the average depth of the water in kilometers. A tsunami is found to be traveling at 145 kilometers per hour. What is the average depth of the water? Round to the nearest hundredth.

Example 3 Graph $y = 2 + \sqrt{x-1}$.

Make a table of values and graph the function.

x	1	2	3	4	5
y	2	3	$2 + \sqrt{2}$ or 3.4	$2 + \sqrt{3}$ or 3.7	7



7-4 *n*th Roots (pp. 402–406)

Simplify.

31. $\pm\sqrt{256}$

32. $\sqrt[3]{-216}$

33. $\sqrt{(-8)^2}$

34. $\sqrt[5]{c^5 d^{15}}$

35. $\sqrt{(x^4-3)^2}$

36. $\sqrt[3]{(512+x^2)^3}$

37. $\sqrt[4]{16m^8}$

38. $\sqrt{a^2 - 10a + 25}$

39. PHYSICS The velocity v of an object can be defined as $v = \sqrt{\frac{2K}{m}}$, where m is the mass of an object and K is the kinetic energy. Find the velocity of an object with a mass of 15 grams and a kinetic energy of 750.

Example 4 Simplify $\sqrt{81x^6}$.

$\sqrt{81x^6} = \sqrt{(9x^3)^2}$ $81x^6 = (9x^3)^2$

$= 9|x^3|$ Use absolute value since x could be negative.

Example 5 Simplify $\sqrt[7]{2187x^{14}y^{35}}$.

$\sqrt[7]{2187x^{14}y^{35}}$

$= \sqrt[7]{(3x^2y^5)^7}$ $2187x^{14}y^{35} = (3x^2y^5)^7$

$= 3x^2y^5$ Evaluate.

Mixed Problem Solving
For mixed problem-solving practice, see page 932.

7–5 Operations with Radical Expressions (pp. 408–414)

Simplify.

40. $\sqrt[6]{128}$

41. $5\sqrt{12} - 3\sqrt{75}$

42. $6\sqrt[5]{11} - 8\sqrt[5]{11}$

43. $(\sqrt{8} + \sqrt{12})^2$

44. $\sqrt{8} \cdot \sqrt{15} \cdot \sqrt{21}$

45. $\dfrac{\sqrt{243}}{\sqrt{3}}$

46. $\dfrac{1}{3 + \sqrt{5}}$

47. $\dfrac{\sqrt{10}}{4 + \sqrt{2}}$

48. GEOMETRY The measures of the legs of a right triangle can be represented by the expressions $4x^2y^2$ and $8x^2y^2$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.

Example 6 Simplify $6\sqrt[5]{32m^3} \cdot 5\sqrt[5]{1024m^2}$.

$6\sqrt[5]{32m^3} \cdot 5\sqrt[5]{1024m^2}$

$= 6 \cdot 5\sqrt[5]{(32m^3 \cdot 1024m^2)}$ Product Property of Radicals

$= 30\sqrt[5]{2^5 \cdot 4^5 \cdot m^5}$ Factor into exponents of 5 if possible.

$= 30\sqrt[5]{2^5} \cdot \sqrt[5]{4^5} \cdot \sqrt[5]{m^5}$ Product Property of Radicals

$= 30 \cdot 2 \cdot 4 \cdot m$ or $240m$ Write the fifth roots.

7–6 Rational Exponents (pp. 415–421)

Evaluate.

49. $27^{-\frac{2}{3}}$

50. $9^{\frac{1}{3}} \cdot 9^{\frac{5}{3}}$

51. $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}}$

Simplify.

52. $\dfrac{1}{y^{\frac{2}{5}}}$

53. $\dfrac{xy}{\sqrt[3]{z}}$

54. $\dfrac{3x + 4x^2}{x^{-\frac{2}{3}}}$

55. ELECTRICITY The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power in watts and R is the resistance in ohms. How much current does an appliance use if $P = 120$ watts and $R = 3$ ohms? Round your answer to the nearest tenth.

Example 7 Write $32^{\frac{4}{5}} \cdot 32^{\frac{2}{5}}$ in radical form.

$32^{\frac{4}{5}} \cdot 32^{\frac{2}{5}} = 32^{\frac{4}{5} + \frac{2}{5}}$ Product of powers

$= 32^{\frac{6}{5}}$ Add.

$= (2^5)^{\frac{6}{5}}$ $32 = 2^5$

$= 2^6$ or 64 Power of a power

Example 8 Simplify $\dfrac{3x}{\sqrt[3]{z}}$.

$\dfrac{3x}{\sqrt[3]{z}} = \dfrac{3x}{z^{\frac{1}{3}}}$ Rational exponents

$= \dfrac{3x}{z^{\frac{1}{3}}} \cdot \dfrac{z^{\frac{2}{3}}}{z^{\frac{2}{3}}}$ Rationalize the denominator.

$= \dfrac{3xz^{\frac{2}{3}}}{z}$ or $\dfrac{3x\sqrt[3]{z^2}}{z}$ Rewrite in radical form.

7–7 Solving Radical Equations and Inequalities (pp. 422–427)

Solve each equation or inequality.

56. $\sqrt{x} = 6$

57. $y^{\frac{1}{3}} - 7 = 0$

58. $(x - 2)^{\frac{3}{2}} = -8$

59. $\sqrt{x + 5} - 3 = 0$

60. $\sqrt{3t - 5} - 3 = 4$

61. $\sqrt{2x - 1} = 3$

62. $\sqrt[4]{2x - 1} = 2$

63. $\sqrt{y + 5} = \sqrt{2y - 3}$

64. $\sqrt{y + 1} + \sqrt{y - 4} = 5$

65. $1 + \sqrt{5x - 2} > 4$

66. $\sqrt{-2x + 14} - 6 \geq -4$

67. $10 - \sqrt{2x + 7} \leq 3$

68. $6 + \sqrt{3y + 4} < 6$

69. $\sqrt{d + 3} + \sqrt{d + 7} > 4$

70. $\sqrt{2x + 5} - \sqrt{9 + x} > 0$

71. **GRAVITY** Hugo drops his keys from the top of a Ferris wheel. The formula $t = \frac{1}{4}\sqrt{65 - h}$ describes the time t in seconds when the keys are h feet above the boardwalk. If Hugo was 65 meters high when he dropped the keys, how many meters above the boardwalk will the keys be after 2 seconds?

Example 9 Solve $\sqrt{3x - 8} + 1 = 3$.

$\sqrt{3x - 8} + 1 = 3$	Original equation
$\sqrt{3x - 8} = 2$	Subtract 1 from each side.
$\left(\sqrt{3x - 8}\right)^2 = 2^2$	Square each side.
$3x - 8 = 4$	Evaluate the squares.
$3x = 12$	Add 8 to each side.
$x = 4$	Divide each side by 3. Check this solution.

Example 10 Solve $\sqrt{4x - 3} - 2 > 3$.

$\sqrt{4x - 3} - 2 > 3$	Original inequality
$\sqrt{4x - 3} > 5$	Add 2 to each side.
$\left(\sqrt{4x - 3}\right)^2 > 5^2$	Square each side.
$4x - 3 > 25$	Evaluate the squares.
$4x > 28$	Add 3 to each side.
$x > 7$	Divide each side by 4.